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ALGEBRAIC EXPRESSIONS AND POLYNOMIALS

- **Constant:** Quantity which has a fixed numerical value e.g. 0, 1, 2
- **Variable:** Quantity which can take different numerical values .A variable is represented by a letter of the English alphabet such as a, b, c, x, y, z etc.
- **Algebraic expressions:** A combination of constants and variables, connected by any or all of the four fundamental operations (+, -, ×, ÷).
- **Term:** Each part of the expression along with its sign
- **Monomial:** An algebraic expression containing one term e.g. $6a^2$, $3x^2y^2$ etc.
- **Binomial:** An algebraic expression containing two terms e.g. $a^2 + b^2$, $7xy + y^2$ etc.
- **Trinomial:** An algebraic expression containing three terms e.g. $x^2 + y^2 + z^2$, $x^2 + 2xy + y^2$ etc.
- **Polynomial:** An algebraic expression in which variable(s) does (do) not occur in the denominator, exponents of variables are whole numbers and numerical coefficients of various terms are real numbers e.g. $x^3 - 2y^2 + y - \sqrt{7}$ is a polynomial while $x^3 - \frac{1}{x}$ is not a polynomial.
- **Factor:** When two or more numbers or variables are multiplied, then each one of them and their product is called a factor of the product. A constant factor is a numerical factor while a variable is known as a literal factor.
- **Coefficient:** In a term any one of the factors with the sign of the term is the coefficient of the product of the other factors e.g. in $-3xy$, coefficient of x is $-3y$.
- **Constant Term:** Term which has no literal factor e.g. in $2x + 9y + 7$ the constant term is 7.
- **Like and Unlike Terms:** Terms having same literal factors are called like or similar terms and terms having different literal factors are called unlike terms.
- **Degree of a polynomial :** Sum of the exponents of the variables in a term is called degree of the term.
Degree of a polynomial is the same as the degree of its term or terms having the highest degree and non-zero coefficient.
- **Quadratic polynomial :** A polynomial of degree 2 e.g. $x^2 - 3x + 2$.
- **Zero degree polynomial:** Degree of a non-zero constant polynomial is taken as zero
Zero polynomial : When all the coefficients of variables in the terms of a polynomial are zeros, the polynomial is called a zero polynomial and the degree of zero polynomial is not defined.
- **Zeros of a polynomial:** Value(s) of the variable for which the value of a polynomial in one variable is zero.
- **Addition and subtraction of polynomials:**
The sum of two (or more) like terms is a like term whose numerical coefficient is the sum of the numerical coefficients of the like terms
The difference of two like terms is a like term whose numerical coefficient is the difference of the numerical coefficients of the like terms
To add polynomials, add their like terms together e.g. $2x + 3x = 5x$, $3x^2y + 8x^2y = 11x^2y$
To subtract a polynomial from another polynomial subtract a term from a like term e.g. $9x^2y^2 - 5x^2y^2 = 4x^2y^2$, $5y - 2y = 3y$.
- **Multiplication of the polynomials:** To multiply a monomial by a monomial, use laws of

exponents and the rules of the signs e.g. $3a \times a^2b^2 = 3a^3b^2$

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial.

To multiply a polynomial by another polynomial multiply each term of the one polynomial by each term of the other polynomial and simplify the result by combining like terms.

- **Division of polynomials:** To divide a monomial by another monomial, find the quotient of numerical coefficients and variables separately

using laws of exponents and then multiply these quotients.

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial.

Process of division of a polynomial by another polynomial is done on similar lines as in arithmetic after arranging the terms of both polynomials in decreasing powers of the variable common to both of them.

If remainder is zero the divisor is a factor of dividend.

Dividend = Divisor \times quotient + Remainder.

CHECK YOUR PROGRESS:

- The degree of a non zero constant is :
(A) 0 (B) 1 (C) 2 (D) 3
- The coefficient of x^5 in $7x^5y^3$ is :
(A) 7 (B) 4^3 (C) $7y^3$ (D) 5
- The degree of the polynomial $5x^6y^4 + x^2y + xy^2 - 3xy + 4$ is:
(A) 2 (B) 3 (C) 6 (D) 10
- Which of the following is a polynomial?
(A) $x^2 - 5\sqrt{x} + 2$ (B) $\sqrt{x} + \frac{1}{\sqrt{x}}$ (C) $\frac{5}{x^2 - 3x + 1}$ (D) None of these
- A zero of the polynomial $x^2 - 2x - 15$ is:
(A) -5 (B) -3 (C) 0 (D) 3
- Which of the following pairs of terms is a pair of like terms?
(A) $2a, 2b$ (B) $2xy^3, 2x^3y$ (C) $3x^2y, \frac{1}{\sqrt{2}}yx^2$ (D) $8, 16a$
- Add $\frac{2}{3}x^2 + x + 1$ and $\frac{3}{7}x^2 + \frac{1}{4}x + 2$.
- Subtract $7x^3 - 3x^2 + 2$ from $x^2 - 5x + 2$.
- Find the product of $(2x + 3)$ and $(x^2 - 3x + 4)$.
- Find the quotient and remainder when $6x^2 - 5x + 1$ is divided by $2x - 1$.
- Evaluate $3xy - x^3 - y^3 + z^3$ at $x = 2, y = 1, z = -3$.

